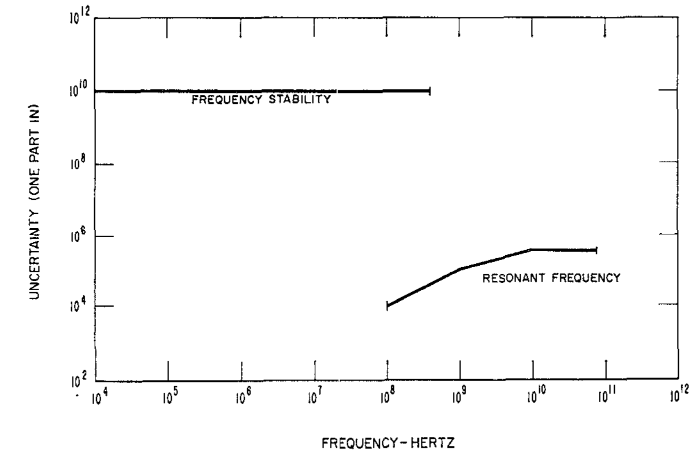

Fig. 1. Microwave and high-frequency calibrations (rectangulra waveguide and coaxial).

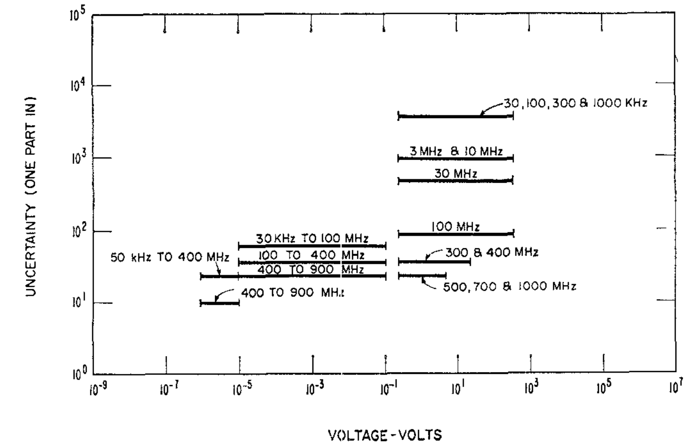

Fig. 2. High-frequency voltage calibrations (coaxial).

having coaxial terminals with Type *N* connectors (male or female) in the frequency range 100 MHz to 10 GHz, and on cavity wavemeters having standard rectangular waveguide terminals in the frequency range 2.6 to 75 GHz.

Item	Description
201.930a	Measurement of resonant frequency of fixed cavity wavemeter.
201.930b	Setting of adjustable cavity wavemeter at prescribed resonant frequency.
201.930c-1	Calibration of dial setting vs resonant frequency of variable cavity wavemeter at initial prescribed frequency.
201.930c-2	Calibration of dial setting vs resonant frequency of variable cavity wavemeter at each prescribed frequency additional to the initial frequency and on the same wavemeter as 201.930c-1.
201.930z	Special calibrations not covered by the above schedule.

High-Frequency Region

201.860 *Frequency stability calibration of signal sources*

Frequency stability calibrations are made on signal sources in the frequency range 30 kHz to 500 MHz. The signal source should have a power output of at least 10 mW (into a matched load). The frequency stability of the signal source should be better than approximately one part in 10^7.

Item	Description
201.860	Measurement of frequency stability of signal sources from 30 kHz to 500 MHz.

201.810 *Thermal converters, RF-dc voltmeters and RF voltmeters*

Ordinarily, instruments which are equally suitable for dc and RF use are calibrated only for RF-dc difference by the procedure of item 201.810a since periodic calibrations can be made by the user with a calibrated dc instrument. Such dc calibrations will be made at NBS only under unusual circumstances and by advance arrangement. Instruments suitable for RF use only are given RF calibrations by the procedures of items 201.810a and 201.810b. Instruments which respond to average or peak values, or which are not in *ASA accuracy class $\frac{1}{4}$ per cent* or better, usually are not accepted for calibration below 30 MHz.

Item	Description
201.810a	Determination of voltage at 30, 100, 300 kHz; 1, 3, 10, 30, and 100 MHz from 0.2 to 300 V.
201.810b	Determination of voltage at 300 and 400 MHz from 0.2 to 20 V, and at 500, 700, and 1000 MHz from 0.2 to 7 V.
201.810z	Special calibrations not covered by the above schedule.

201.811 *Radio-frequency voltmeters and signal sources*

Normally, NBS accepts for calibration only high-quality voltmeters suitable for use as interlaboratory standards. These instruments should have a stability of one per cent or better, and an accuracy of three per cent or better. Voltmeters are calibrated by the procedures of item 201.811a and 201.811b. NBS usually accepts only signal sources (signal generators) of sufficiently high quality to be considered as interlaboratory standards. If these instruments are equally suitable for dc and RF use, they are calibrated for RF-dc difference by the procedures of item 201.811a, 201.811b, and 201.811c. Signal sources suitable for RF use only are calibrated by the procedures of items 201.811a and 201.811c.

Item	Description
201.811a	Determination of voltage for voltmeters and of RF micropotentiometers and signal sources from 30 kHz to 400 MHz, from 1 μV to 0.1 V.
201.811b	Determination of voltage for voltmeters from 400 to 1000 MHz, from 100 μV to 0.1 V.
201.811c	Determination of voltage of RF micropotentiometers and signal sources from 30 kHz to 900 MHz, from 1 μV to 0.1 V.
201.811z	Special calibrations not covered by the above schedule.

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Comment on "Wave Propagation in Sinusoidally Stratified Dielectric Media"

Great interest has been evidenced in a recent paper by Tamir and his associates[1] with regard to the problems involved in solving the equations which govern wave propagation in a waveguide containing an inhomogeneous dielectric media with a sinusoidal variation of the permittivity in the logitudinal direction. In this communication, an alternative approach to the solution of the equations is presented in which the vector wave equation method, as described by Hansen [1] and Stratton [2], is followed. In conjunction with this approach, the Hertzian vector technique is employed in a particularly appealing and satisfying manner. The usefulness of the methodology described is more than adequately demonstrated.

Manuscript received July 30, 1964.

[1] Tamir, T., et al., *IEEE Trans. Microwave Theory and Techniques*, vol MTT-12, May 1964, pp 323–335.

After the equations have been derived, some remarks concerning possible application of waveguides filled with sinusoidally varying dielectric media are presented.

The vector wave equations needed to describe the problem are readily formulated. It is assumed that the z-axis is the longitudinal coordinate with the permittivity represented as some $\epsilon(z)$. It is also assumed that the suppressed time dependence of the field vectors $\boldsymbol{E}$ and $\boldsymbol{H}$ is of the form $e^{-i\omega t}$. Maxwell's curl equations for a source free region therefore, are

$$\nabla \times \boldsymbol{E} = i\omega\mu_0 \boldsymbol{H} \tag{1}$$

and

$$\nabla \times \boldsymbol{H} = -i\omega\epsilon(z)\boldsymbol{E} \tag{2}$$

These equations yield the following vector wave equations in E and H, respectively:

$$\nabla \times \nabla \times \boldsymbol{E} - \omega^2\mu_0\epsilon(z)\boldsymbol{E} = 0 \tag{3}$$

and

$$\nabla \times \nabla \times \boldsymbol{H} - \frac{\nabla\epsilon(z)}{\epsilon(z)} \times \nabla \times \boldsymbol{H} - \omega^2\mu_0\epsilon(z)\boldsymbol{H} = 0 \tag{4}$$

Equations (3) and (4) are taken as the point of departure for our discussion, and it is here that the Hertzian vector technique is applied. If scalar functions $\phi(x, y, z)$ and $\psi(x, y, z)$ are introduced, then the field vectors representing transverse electric (TE) and transverse magnetic (TM) waveguide modes admit of the following representation.

TE modes are symbolized by subscript *te*:

$$\boldsymbol{E}_{te} = \nabla \times [\phi(x, y, z)\hat{a}_z] \tag{5}$$

and

$$\boldsymbol{H}_{te} = -\frac{i}{\omega\mu_0} \nabla \times \nabla \times [\phi(x, y, z)\hat{a}_z] \tag{6}$$

where, $\hat{a}_z$ represents the unit vector in the z direction.

TM modes are symbolized by subscript *tm*:

$$\boldsymbol{H}_{tm} = \nabla \times [\psi(x, y, z)\hat{a}_z] \tag{7}$$

and

$$\boldsymbol{E}_{tm} = \frac{i}{\omega\epsilon(z)} \nabla \times \nabla \times [\psi(x, y, z)\hat{a}_z] \tag{8}$$

It is noted, in passing, that (5)–(8) do in fact satisfy Maxwell's divergence equations.

By substitution of (5) into (3), and (7) into (4), the following vector equations obtain:

$$\nabla \times \nabla \times \nabla \times [\phi(x, y, z)\hat{a}_z] - \omega^2\mu_0\epsilon(z)\nabla \times [\phi(x, y, z)\hat{a}_z] = 0 \tag{9}$$

and similarly

$$\nabla \times \nabla \times \nabla \times [\psi(x, y, z)\hat{a}_z] - \frac{\nabla\epsilon(z)}{\epsilon(z)} \times \nabla \times \nabla \times [\psi(x, y, z)\hat{a}_z] - \omega^2\mu_0\epsilon(z)\Delta \times [\psi(x, y, z)\hat{a}_z] = 0 \tag{10}$$

Fortunately (9) and (10) are amenable to a method of separation of variables, in which case $\phi(x, y, z)$, and $\psi(x, y, z)$ become, written in general form:

$$\phi(x, y, z) = \begin{bmatrix}\sin \\ \cos\end{bmatrix} p_{te}x \begin{bmatrix}\sin \\ \cos\end{bmatrix} q_{te}y \cdot \begin{bmatrix}T_1 \\ T_2\end{bmatrix}(z) \tag{11}$$

and

$$\psi(x, y, z) = \begin{bmatrix}\sin \\ \cos\end{bmatrix} p_{tm}x \begin{bmatrix}\sin \\ \cos\end{bmatrix} q_{tm}y \cdot \begin{bmatrix}U_1 \\ U_1\end{bmatrix}(z) \tag{12}$$

where separation constants for the transverse electric, magnetic modes are, respectively: p_{te}, q_{te}; p_{tm}, q_{tm}.

The functions $T_{1,2}(z)$ and $U_{1,2}(z)$ are solutions of the following, differential equations:

$$[(d^2/dz^2) + \omega^2\mu_0\epsilon(z) - p_{te}^2 - q_{te}^2]T_{1,2}(z) = 0 \tag{13}$$

and

$$\left[\frac{d^2}{dz^2} + \frac{1}{\epsilon(z)}\frac{\partial\epsilon(z)}{\partial z}\frac{d}{dz} + \omega^2\mu_0\epsilon(z) - p_{tm}^2 - q_{tm}^2\right]U_{1,2}(z) = 0 \tag{14}$$

It is clear, therefore, that selection of the functional behavior of $\epsilon(z)$ will govern the solutions of (13) and (14). For the case to be considered, we shall assume the following representation of the permittivity:

$$\epsilon(z) = \epsilon_r[A - 2B\cos^2 cz] \tag{15}$$

If it is realized that

$$\cos^2\theta \equiv \tfrac{1}{2}[1 + \cos 2\theta] \tag{16}$$

then substitution of (16) into (15), and comparison of (16) with (1) arrived at by Tamir, et al. yields the following identification of constants:

$$\epsilon(z) = \epsilon_a[(A - B) - B\cos 2cz] \tag{17}$$

where $cz = \theta$

$$\left.\begin{aligned} A &= M + 1 \\ B &= M \\ C &= \frac{\pi}{d} \end{aligned}\right\} \tag{18}$$

Therefore, (18) into (17) yields

$$\epsilon(z) = \epsilon_r[1 - M\cos(2\pi z/d)] \tag{18a}$$

But, (18a) is but Tamir, et al., eq. (1), so that the identification is complete.

Substitution of (15) into (13) and (14), respectively, yeilds the following:

$$[(d^2/dz^2) + \omega^2\mu_0\epsilon_r(A - 2B\cos^2 cz) - p_{te}^2 - q_{te}^2]T_{1,2}(z) = 0 \tag{19}$$

and

$$\left\{\frac{d^2}{dz^2} + \left[\frac{\cos cz \sin cz}{\frac{A}{4B} - \frac{1}{2}\cos^2 cz}\right]\frac{d}{dz} + \omega^2\mu_0\epsilon_r[A - 2B\cos^2 cz] - p_{tm}^2 - q_{tm}^2\right\}U_{1,2}(z) = 0. \tag{20}$$

Now, introducing the dimensionless variable $u = zc$, and noting that

$$\left.\begin{aligned} \frac{d}{du} &= \frac{1}{c}\frac{d}{dz} \\ \frac{d^2}{du^2} &= \frac{1}{c^2}\frac{d^2}{dz^2} \end{aligned}\right\} \tag{21}$$

(19) and (20) become (22) and (23), respectively

$$\left[\frac{d^2}{du^2} + \frac{1}{c^2}(\omega^2\mu_0\epsilon_r A - p_{te}^2 - q_{te}^2) - \frac{2\omega^2\mu_0\epsilon_r B}{c^2}\cos^2 u\right]T_{1,2}(z) = 0 \tag{22}$$

$$\left\{\frac{d^2}{du^2} + \frac{1}{c}\left[\frac{\cos u \sin u}{\frac{A}{4B} - \frac{1}{2}\cos^2 u}\right]\frac{d}{du} + \frac{1}{c^2}(\omega^2\mu_0\epsilon_r A - p_{tm}^2 - q_{tm}^2) - \frac{2\omega^2\mu_0\epsilon_r B}{c^2}\cos^2 u\right\}U_{1,2}(z) = 0 \tag{23}$$

Thus (22) is immediately recognized as Mathieu's Equation (3) in canonical form; (23) also can be expressed in this form if it is assumed that the term

$$\left[\frac{\partial\epsilon(z)}{\partial z}\frac{d}{dz}U_{1,2}(z)\right]$$

is negligible compared to the other terms in (14).

Therefore, solutions for the TE modes may be obtained directly from a consideration of (5) (6), (11), and (22). Similarly, solutions for the TM modes can be derived from consideration of (7), (8), (12), and (23) provided the restriction on the first derivative term (as noted in the foregoing) obtains. The author is currently studying methods of solution of (23) without this restrictive assumption and hopes that results will be available shortly.

Considering (22), it is noted that owing to the stability properties of the Mathieu functions, the presence of stop and pass bands is indicated, the boundaries between stable and unstable solutions indicate the wave cutoff frequencies. Furthermore, for a rectangular guide filled with the nonhomogeneous medium described by (15), the separation constants p_{te} and q_{te} are the usual mode indices as given by

$$p_{te}(n) = \frac{\pi h}{a}, \qquad (n = 0, 1, \cdots)$$

and

$$q_{te}(m) = \frac{\pi m}{b}, \qquad (m = 0, 1, \cdots)$$

If we consider wave propagation in a corrugated waveguide [4], the similarities between that case and the present one are clear. It may, therefore, be possible to replace a corrugated waveguide with one containing a sinusoidally varying dielectric. Additionally, the presence of pass bands indiates that by varying the frequency of an ultrasonic standing wave in the dielectric, for example, either a tunable band-pass filter or modulator may be realized, provided a suitable pressure sensitive dielectric is available.

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REFERENCES

[1] Hansen, W. W., *Phys. Rev.*, vol 47, 1935, p 139.
[2] Stratton, J. A., *Electromagnetic Theory*. New York: McGraw-Hill, 1941.
[3] Morse, P. M., and H. Feshbach, *Methods of Theoretical Physics*, pt I. New York: McGraw-Hill, 1953.
[4] Brillouin, L., *Wave Propagation in Periodic Structures*, 2nd ed. New York: Dover Pub., 1953.

Authors' Comment[2]

The communication by Kallas presents an alternative derivation of the fundamental differential equations on which we base the various developments in our paper. His procedure follows the classical form which employs Hertz vectors, but it results in a more cumbersome derivation than ours is. We used the modal formulation,[3] which permits an immediate reduction to a scalar form and the elimination of the superfluous transverse dependences. Thus, we feel that our procedure is simpler, but we agree that some may prefer the classical one and that, possibly, it is a matter of background and taste. With either derivation, of course, one recognizes that at that stage the problem has only been set up, and that the bulk of the work, involving the solution to the equations and its interpretation in physical problems, still lies ahead.

The waveguide case, to which Kallas refers, was also treated in Section III-C of our work. We did not treat the more difficult Hill's equation, which applies for E or TM modes. We wish Kallas well in his efforts in this direction.

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[2] Manuscript received September 22, 1964.
[3] Marcuvitz, N., *Waveguide Handbook*, Rad. Lab. Ser., vol 10. New York: McGraw-Hill, 1951, ch 1.

CORRECTIONS

Wave Propagation in Sinusoidally Statified Dielectric Media[1]

On pp. 331–332 the term $\kappa_u d/\pi$ should be replaced by its inverse, i.e., $\pi/\kappa_u d$, in the numerator only of (50), (54), and (55).

The paragraph immediately following (55) should read: "when $\epsilon_1 = \epsilon_r$. At low frequencies, where $\kappa_u d$ is appreciably smaller than π, the reflection from the interface in Fig. 11(a) is seen to be less than that for Fig. 11(b) because the front quarter-sine-wave section in Fig. 11(a) acts as a matching section. At high frequencies, at which $\kappa_u d$ is much larger than π, the interface discontinuity becomes dominant, and the structure of Fig. 11(a) produces the larger reflection coefficient."

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Manuscript received Nov. 9, 1964.
[1] Tamir, T., H. C. Wang, and A. A. Oliner, *IEEE Trans. on Microwave Theory and Techniques*, vol MTT-12, no 3, May 1964, pp 323–335.

Addendum to: An Exact Method for Synthesis of Microwave Band-Stop Filters[1]

The title of Table VII (page 380) should read:

> h Values for 0.01-dB Ripple Chebyshev Filters Having h_0 and $h_{n+1} = 1.0$ and Various Stop-Band 0.01-dB Fractional Bandwidths w.

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Manuscript received August 19, 1964.
[1] Cristal, E. G., *IEEE Trans. on Microwave Theory and Techniques (Correspondence)*, vol MTT-12, May 1964, pp 369–382.

Coupled Circular Cylindrical Rods Between Parallel Ground Planes[1]

Page 429, (5) should read:

$$C_m = 1/4(C_0 - C_e).$$

Page 434, last line of footnote 2 should read:

$$(3/4)V(\bar{r}_0).$$

Footnote 3 should read:

> [3] Equation (7) may be derived by taking the two-dimensional integral formulation for the potential at a point within a closed boundary and permitting the observation point to approach the boundary (see, for example, J. A. Stratton, "Electromagnetic Theory," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 166–170 and problem 8, p. 219; 1941). The limiting operation may be performed as in Mai and Van Bladel [11], or alternatively, the observation point may be placed on the boundary and the boundary curve in the vicinity of the observation point deformed into an appropriate segment of a circle of infinitesimally small radius. The deformation is such as to leave the observation point within the boundary. The segment of circle is π radians where the boundary curve has a unique tangent and is $\frac{3}{2}\pi$ radians at the right angle.

Page 435, Equation (9): the summation should be over the index j rather than i. Eq. (10): the last entry of the η/ϵ column vector should read η_n/ϵ.

Page 437, Equation (14): the summation should be over the index m rather than n.

Page 439, Section VII, 2nd column, lines 2 and 3: the phrase "rectangular bears" should read "rectangular bars."

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Manuscript received Aug. 19, 1964.
[1] Cristal, E. G., *IEEE Trans. on Microwave Theory and Techniques*, vol MTT-13, Jul 1964, pp 428–439.